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Energy relaxation due to TO phonon emission in polar semiconductors

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Abstract. Electron transitions involving the emission of transverse optical (TO) phonons in polar semiconductors are discussed. Distinction is made in particular between the contributions from the TO modes to the optical deformation potential interaction and the interaction of the TO modes arising via their electromagnetic vector potential. The latter is due solely to the polar nature of the semiconductor. A theory for the emission rate Γ_{TO} due to the interaction of electrons with the vector potential is described and the calculated rate is compared with the corresponding rate Γ_{Def} involving the optical deformation potential and with Γ_{LO}, the emission rate arising from the well known Fröhlich mechanism due to the polar longitudinal optical (LO) modes. Typical values are obtained for the case of GaAs in the homogeneous bulk.

1. Introduction

The recent surge in research activity constituting the field of low-dimensional structures (LDS) is partly motivated by the possibility of fabricating faster and smaller electronic devices, e.g. those based on III–V compounds. Of considerable interest in this context, therefore, is the transition rate for carrier-energy relaxation via the various possible channels (see, e.g., Conwell 1967, Ridley 1988). A great deal of effort is devoted to the evaluation of such relaxation rates, both experimentally and theoretically (for an account see, e.g., Shah 1987). Like other electronic attributes of LDS, the relevant theoretical work here requires first an examination of how the theory in the homogeneous bulk can be suitably adapted to deal with the cases in low dimensions.

In a non-polar semiconductor the optical phonons, both transverse (TO) and longitudinal (LO), interact with the carriers via the so-called optical deformation potential interaction (see, e.g., Ridley 1988). In polar semiconductors there are additional effects due to the polar nature of the semiconductor which, at low energies, constitute the dominant influence on the carrier properties. The LO phonons exhibit Coulomb fields which manifest themselves in the coupling to free carriers via the Fröhlich interaction. Less well known are the corresponding electromagnetic properties of the TO modes, as well as their influence on the carrier properties in polar materials. These issues constitute the main concern of this paper, particularly with reference to III–V compounds such as GaAs.

In III–V compounds at room temperature, the emission of LO phonons is indeed regarded as the main channel of energy relaxation for carriers in the homogeneous

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bulk. These longitudinal modes have Coulomb fields whose influence on the carriers is envisaged to be much stronger than that due to the electromagnetic effects arising from the transverse optical (TO) counterparts. The electromagnetic interactions of the TO modes have accordingly been ignored, except for the high energies (Fröhlich 1937, Callen 1949) at which the optical deformation potential interaction becomes significant. It appears, however, that there is at present no quantitative analysis of the comparison between the emission rates arising via the LO and the TO channels in the same polar material. As we pointed out above, such an analysis is an essential prerequisite of the corresponding treatments of energy relaxation in the cases of low dimensions.

A considerable amount of work has already been done on the theory of LO phonon emission in quantum wells and superlattices (Ridley 1982, Riddoch and Ridley 1984, Lassnig 1984, Sawaki 1986, Babiker and Ridley 1986, Babiker *et al* 1987, 1988, Wendler and Bechstedt 1987). The corresponding experimental work is gathering momentum (Devand *et al* 1989, Westland *et al* 1988, Abstreiter *et al* 1988, Seilmeier *et al* 1987). At present it appears that there is no clear quantitative picture of energy relaxation in the various situations in LDS. One of the theoretical predictions, however, is that, besides electron and phonon resonances, the LO phonon rate decreases with decreasing well width in quantum wells and superlattices (Ridley 1987, Babiker *et al* 1987, 1988). For very narrow wells and for free standing wafers and wires (Leburton 1984, Degani and Hipolito 1988, Cibert *et al* 1986) the LO phonon rates are expected to be considerably diminished relative to values in the homogeneous bulk. The corresponding electromagnetic attributes of the TO modes are also expected to suffer changes relative to the case in the bulk which, as we have emphasised, has not yet been explored.

The purpose of this paper is therefore to take the first essential steps by developing the theory for electron interactions involving the electromagnetic properties of the TO modes, and we apply the theory to GaAs as a typical III–V compound. It is our intention in future reports to extend the theory to the important two- and one-dimensional cases mentioned above. The paper is organised as follows. In § 2 we write down the field equations governing the TO vibrations in a bulk polar semiconductor, and derive the corresponding free-field Hamiltonian from an energy flow theorem. This is followed in § 3 by the quantisation of the TO modes. In § 4 we write down the total Hamiltonian including the coupling of electrons to the vector potential of the TO modes and outline the calculations for the transition rate using the golden rule. We also exhibit the comparison between the TO and the optical deformation interaction as well as the LO interaction for the case of electrons and we explore the general results by application to the case of GaAs. Section 5 contains conclusions and further comments.

2. Basic theory

The transverse optical lattice vibrations in polar semiconductors are synonymous with phonon polaritons (see, e.g., Mills and Burstein 1974). The basic theory can conveniently be described using the hydrodynamic continuum treatment by Born and Huang (1968). Here we follow the extended version of the Born and Huang treatment as given by Babiker (1986) including dispersion. The equations for the TO modes can be separated from those for the LO modes and can be written as follows

$$\nabla \cdot \boldsymbol{E} = 0 = \nabla \cdot \boldsymbol{B} \tag{2.1}$$

$$\boldsymbol{D} = \boldsymbol{E} + 4\pi \boldsymbol{P} \tag{2.2}$$

$$\nabla \times \boldsymbol{E} = -\dot{\boldsymbol{B}}/c \qquad \nabla \times \boldsymbol{B} = (1/c)\dot{\boldsymbol{D}} \tag{2.3}$$

$$\ddot{\boldsymbol{u}} = -\omega_T^2 \boldsymbol{u} + g\boldsymbol{E} - \beta^2 \nabla^2 \boldsymbol{u} \tag{2.4}$$

$$\boldsymbol{P} = g\boldsymbol{u} + [(\varepsilon_\infty - 1)/4\pi]\boldsymbol{E} \tag{2.5}$$

where

$$g = [\omega_T^2(\varepsilon_0 - \varepsilon_\infty)/4\pi]^{1/2}. \tag{2.6}$$

Here *all fields are transverse* (i.e. have zero divergence, as in equation (2.1)). $\boldsymbol{E}$ and $\boldsymbol{B}$ are the transverse electric and magnetic fields associated with the TO vibrations. $\boldsymbol{P}$ is the transverse polarisation field. $\boldsymbol{D}$ is the corresponding electric displacement field and $\boldsymbol{u}$ is the transverse ionic amplitude field, as defined by Born and Huang (1968). The parameters ε_0, ε_∞, ω_T and β characterise the polar material with ε_0 and ε_∞ the static and high-frequency dielectric constants, ω_T is the limiting transverse optical frequency and β is typically an acoustic velocity, i.e. such that $\beta \ll c$ with c the velocity of light *in vacuo*.

The field equation follows directly from the two equations in (2.3). We have

$$\nabla \times \nabla \times \boldsymbol{E} = -\ddot{\boldsymbol{D}}/c^2. \tag{2.7}$$

We assume the spacetime dependence $\exp(i\boldsymbol{k}\cdot\boldsymbol{x} - i\omega t)$ for a mode of frequency ω and wavevector $\boldsymbol{k}$. Then from equations (2.2), (2.4) and (2.5) we have, by eliminating $\boldsymbol{u}$ and $\boldsymbol{P}$ in favour of $\boldsymbol{E}$, that

$$\boldsymbol{D} = \varepsilon(\boldsymbol{k}, \omega)\boldsymbol{E} \tag{2.8}$$

where

$$\varepsilon(\boldsymbol{k}, \omega) = \varepsilon_\infty + (\varepsilon_0 - \varepsilon_\infty)\omega_T^2/(\omega^2 - \omega_T^2 + \beta^2 k^2). \tag{2.9}$$

We therefore find from (2.7) with (2.1)

$$\nabla^2 \boldsymbol{E} - (\omega^2 \varepsilon(\boldsymbol{k}, \omega)/c^2)\boldsymbol{E} = 0 \tag{2.10}$$

which is of course the well known field equation for all types of polaritons; different polariton species are obtained by changing $\varepsilon(\boldsymbol{k}, \omega)$. The corresponding dispersion relation

$$k^2 c^2/\omega^2 = \varepsilon(\boldsymbol{k}, \omega) \tag{2.11}$$

is the familiar polariton dispersion relation (see, e.g., Mills and Burstein 1974). Note however that with the $\varepsilon(\boldsymbol{k}, \omega)$ given by equation (2.9) this is not the usual phonon–polariton dispersion relation because of the $\beta^2 k^2$ term in the denominator of the second term. As we see below, the presence of this $\beta^2 k^2$ term (due to the inclusion of spatial dispersion terms) accounts for the observed decrease of TO frequency with wavevector at large $k \gg \omega_T/c$.

The dispersion relation is illustrated in figure 1 by plotting ω/ω_T against kc/ω_T. The horizontal axis is on a logarithmic scale and this tends to exaggerate the gradients for small and large kc/ω_T. Note in particular that for large $k \gg \omega_T/c$ (as shown in the inset to figure 1) the dispersion relation is effectively given by

$$\omega^2 = \omega_T^2 - \beta^2 k^2. \tag{2.12}$$

This is the region normally explored for TO dispersion and it is clear that the presence of the $\beta^2 k^2$ term (originating in equation (2.4)) accounts for this experimental fact.

Associated with the electromagnetic fields and polarisation currents (obeying the set

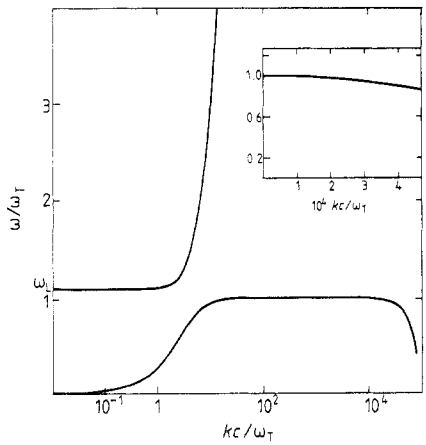

Figure 1. Polariton dispersion curves for GaAs. The parameters are $\omega_T \simeq 270\ \text{cm}^{-1}$, $\beta \simeq 5 \times 10^5\ \text{cm s}^{-1}$, $\varepsilon_0 \simeq 12.93$, $\varepsilon_\infty \simeq 10.9$ and the plot is of ω/ω_T against ck/ω_T. The region of large k is shown by the inset.

of field equations (2.1) to (2.5)) is an energy conservation relation of the form

$$-(\partial/\partial t)[(1/8\pi)(E^2 + B^2)] = \boldsymbol{J}\cdot\boldsymbol{E} + \boldsymbol{\nabla}\cdot[(c/4\pi)(\boldsymbol{E}\times\boldsymbol{B})] \tag{2.13}$$

with

$$\boldsymbol{J} = \dot{\boldsymbol{P}} \tag{2.14}$$

as the transverse current. This is the well known Poynting theorem giving the rate of decrease in energy density as equal to an energy-loss term $\boldsymbol{J}\cdot\boldsymbol{E}$ plus the divergence of an energy flux in the form of the Poynting vector (see, e.g., Landau and Lifshitz 1975). For a mode of frequency ω and wavevector $\boldsymbol{k}$ we have from (2.2) and (2.8) that

$$\boldsymbol{J} = \dot{\boldsymbol{P}} = [(\varepsilon(k,\omega) - 1)/4\pi]\dot{\boldsymbol{E}}. \tag{2.14a}$$

Thus (2.13) becomes

$$-(\partial/\partial t)[(1/8\pi)(\varepsilon(\boldsymbol{k},\omega)E^2 + B^2)] = \boldsymbol{\nabla}\cdot[(c/4\pi)(\boldsymbol{E}\times\boldsymbol{B})] \tag{2.15}$$

and we can write

$$(\partial\mathcal{H}(\boldsymbol{k},\omega)/\partial t) + \boldsymbol{\nabla}\cdot\mathcal{F}(\boldsymbol{k},\omega) = 0 \tag{2.16}$$

where $\mathcal{H}(\boldsymbol{k},\omega)$ and $\mathcal{F}(\boldsymbol{k},\omega)$ are, respectively, the energy density and the Poynting vector associated with the mode of frequency ω and wavevector $\boldsymbol{k}$. Explicitly we write

$$\mathcal{H}(\boldsymbol{k},\omega) = (1/8\pi)(\varepsilon(\boldsymbol{k},\omega)E^2 + B^2) \tag{2.17}$$

with $\varepsilon(\boldsymbol{k},\omega)$ given by (2.9) and

$$\mathcal{F}(\boldsymbol{k},\omega) = (c/4\pi)(\boldsymbol{E}\times\boldsymbol{B}). \tag{2.18}$$

Thus we are able to derive the Hamiltonian density for the polaritons without recourse to the conventional procedure which normally starts from a Lagrangian. The next steps require (2.17) for the quantisation of the TO polaritons in the homogeneous bulk.

3. Field quantisation for TO

All transverse electromagnetic fields (typified by photons in free space) are characterised by two degrees of freedom which are made manifest via the wave polarisation vectors $\hat{\boldsymbol{e}}^{(1)}$ and $\hat{\boldsymbol{e}}^{(2)}$ for each $\boldsymbol{k}$. The free TO field Hamiltonian can now be written in the form

$$H_{\mathrm{f}}^{0} = \sum_{\lambda=1,2} \int \mathrm{d}^3k \int \mathscr{H}^{(\lambda)}(\boldsymbol{k}, \omega, \boldsymbol{x})\, \mathrm{d}^3x \tag{3.1}$$

where, in view of (2.17)

$$\mathscr{H}^{(\lambda)}(\boldsymbol{k}, \omega, \boldsymbol{x}) = (1/8\pi)(\varepsilon(\boldsymbol{k}, \omega)\boldsymbol{E}^{(\lambda)\,2} + \boldsymbol{B}^{(\lambda)2}) \tag{3.2}$$

with $\boldsymbol{E}^{(\lambda)}$ and $\boldsymbol{B}^{(\lambda)}$ denoting the electric and magnetic fields corresponding to the mode of wave polarisation λ, wavevector $\boldsymbol{k}$ and frequency ω. The spatial dependence in $\boldsymbol{E}^{(\lambda)}$ and $\boldsymbol{B}^{(\lambda)}$ has been suppressed for convenience.

The system of transverse fields can be expressed in terms of the transverse vector potential $\boldsymbol{A}^{(\lambda)}$ such that

$$\boldsymbol{E}^{(\lambda)} = -(1/c)\dot{\boldsymbol{A}}^{(\lambda)} \qquad \boldsymbol{B}^{(\lambda)} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(\lambda)}. \tag{3.3}$$

Thus field quantisation can proceed by writing

$$\boldsymbol{A}^{(\lambda)} = C_k^{(\lambda)} \hat{\boldsymbol{e}}^{(\lambda)} (a_k^{(\lambda)}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{HC}) \tag{3.4}$$

where $C_k^{(\lambda)}$ is the mode amplitude to be determined by the usual procedure. The set of orthogonal unit vectors $\hat{\boldsymbol{e}}^{(1)}$, $\hat{\boldsymbol{e}}^{(2)}$ and $\hat{\boldsymbol{k}}$ (with carets denoting unit vectors) must satisfy the following relations

$$\hat{\boldsymbol{e}}^{(\lambda)} \cdot \hat{\boldsymbol{k}} = 0 \qquad \hat{\boldsymbol{e}}^{(\lambda)} \cdot \hat{\boldsymbol{e}}^{(\lambda')} = \delta_{\lambda\lambda'} \tag{3.5}$$

$$\hat{\boldsymbol{e}}^{(1)} \times \hat{\boldsymbol{e}}^{(2)} = \hat{\boldsymbol{k}} \ldots \text{ etc} \tag{3.6}$$

and the following sum rule (see, e.g., Power 1964)

$$\sum_{\lambda=1,2} e_i^{(\lambda)} e_j^{(\lambda)} = \delta_{ij} - \hat{k}_i \hat{k}_j \tag{3.7}$$

where i and j denote Cartesian components. Finally the operators $a_k^{(\lambda)}$ and $a_k^{(\lambda)\dagger}$ are Boson operators satisfying the commutation relations

$$[a_k^{(\lambda)}, a_{k'}^{(\lambda')\dagger}] = \delta_{\lambda\lambda'}\delta(\boldsymbol{k} - \boldsymbol{k}'). \tag{3.8}$$

The mode amplitudes $C_k^{(\lambda)}$ are fixed by the requirement that (dropping zero-point energies)

$$H_{\mathrm{f}}^{0} \rightarrow \sum_{\lambda=1,2} \int \mathrm{d}^3k\, \hbar\omega a_k^{(\lambda)\dagger} a_k^{(\lambda)}. \tag{3.9}$$

We find straightforwardly for the field amplitude

$$C_k^{(\lambda)} = (\hbar\omega/4\pi^2k^2)^{1/2}. \tag{3.10}$$

Thus the total vector potential operator associated with the TO phonon polaritons is given by

$$\boldsymbol{A}(\boldsymbol{x}) = \sum_{\lambda=1,2} \int \mathrm{d}^3\boldsymbol{k} \left(\frac{\hbar\omega}{4\pi^2k^2}\right)^{1/2} \hat{\boldsymbol{e}}^{\lambda} (a_k^{(\lambda)}\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}} + \mathrm{HC}) \tag{3.11}$$

provided that

$$\omega^2 \varepsilon(\boldsymbol{k}, \omega) = c^2 k^2 \tag{3.12}$$

with $\varepsilon(\boldsymbol{k}, \omega)$ given by (2.9). The essential unperturbed field states on which the vector potential operates are written $|\{\boldsymbol{k}, \lambda\}\rangle$, representing one TO polariton of wavevector $\boldsymbol{k}$ and polarisation λ. The no-polariton state is written as $|0\rangle$. These are eigenstates of H_f^0, so we can write

$$H_f^0 |\{\boldsymbol{k}, \lambda\}\rangle = \hbar\omega(k) |\{\boldsymbol{k}, \lambda\}\rangle \tag{3.13}$$

with ω given by the solutions of (3.12).

4. Coupling to electrons

We describe the conduction electrons in polar materials via the effective-mass approximation assuming parabolic bands. Their coupling to the polaritons manifests itself by the usual minimal-coupling prescription involving the vector potential $\boldsymbol{A}$. The (electron + TO field) Hamiltonian including the interaction is thus given by

$$H = [\boldsymbol{P} - (e/c)\boldsymbol{A}(\boldsymbol{x})]^2/2m^* + H_f^0 \tag{4.1}$$

where H_f^0 is given by (3.9) and m^* is the effective mass for electrons in the polar material. $\boldsymbol{P}$ and $\boldsymbol{x}$ are the electron momentum and position operators satisfying the usual commutation relations

$$[P_i, x_j] = -i\hbar\delta_{ij}. \tag{4.2}$$

We can write from (4.1)

$$H = H_0 + H_{int} \tag{4.3}$$

where H_0 is the zero-order Hamiltonian of the two subsystems (electrons + TO field), i.e.

$$H_0 = P^2/2m^* + H_f^0 \tag{4.4}$$

and H_{int} is the interaction Hamiltonian

$$H_{int} = -(e/m^*c)\boldsymbol{A}(\boldsymbol{x}) \cdot \boldsymbol{P} + (e^2/2m^*c^2)A^2(\boldsymbol{x}) \tag{4.5}$$

where we have made use of the fact that $\boldsymbol{\nabla} \cdot \boldsymbol{A} = 0$, as is appropriate for TO modes.

The unperturbed, i.e. zero-order, states of the system satisfy the eigenrelations

$$H_0 |f; \{\boldsymbol{k}, \lambda\}\rangle = (E_f + \hbar\omega(\boldsymbol{k}))|f; \{\boldsymbol{k}, \lambda\}\rangle \tag{4.6}$$

$$H_0 |i; 0\rangle = E_i |i, 0\rangle \tag{4.7}$$

where $|f\rangle$ and $|i\rangle$ are electronic states of energies E_f and E_i, respectively, while $|\{\boldsymbol{k}, \lambda\}\rangle$ stands for a TO field state containing one mode of polarisation λ and wavevector $\boldsymbol{k}$. $|0\rangle$ is the field vacuum.

To leading order in the electron–TO coupling the term $H' = -e\boldsymbol{A} \cdot \boldsymbol{P}/m^*c$ in (4.5) will then effect transitions between the states described by (4.6) and (4.7). The total transition rate from the initial state $|i; 0\rangle$ to all states $|f, \{\boldsymbol{k}, \lambda\}|$ involving the emission of TO modes

is given by

$$\Gamma_{\mathrm{TO}}(E_i) = (2\pi/\hbar) \sum_{\lambda=1,2} \int \mathrm{d}^3 p_f \int \mathrm{d}^3 k \, |\langle i; 0|H_{\mathrm{int}}|f; \{\boldsymbol{k}, \lambda\}\rangle|^2 \times \delta(E_i - E_f - \hbar\omega(k)) \tag{4.8}$$

where $E_i = P_i^2/2m^*$ and the vector potential $\boldsymbol{A}(\boldsymbol{x})$ is given by (3.11) with $\omega(\boldsymbol{k})$ given by (3.12). The matrix element in (4.8) can be evaluated straightforwardly and yields

$$\langle i; 0| - (e/m^*c)\boldsymbol{A}\cdot\boldsymbol{P}|f; \{\boldsymbol{k}, \lambda\}\rangle = -(e/m^*c)(\hbar\omega/4\pi^2k^2)^{1/2}\,\delta(\boldsymbol{P}_i - \boldsymbol{P}_f - \hbar\boldsymbol{k})\hat{\boldsymbol{e}}^{(\lambda)}\cdot\boldsymbol{P}_i \tag{4.9}$$

where we have made use of the transversality condition (3.5). On substituting from (4.9) into (4.8) and carrying out the integral over $\boldsymbol{P}_f$ we find

$$\Gamma_{\mathrm{TO}}(E_i) = \frac{e^2}{2\pi m^{*2}c^2} \int \mathrm{d}^3 k \, \frac{\omega}{k^2}\, \delta\left(\frac{\hbar\boldsymbol{k}\cdot\boldsymbol{P}_i}{m^*} - \frac{\hbar^2k^2}{2m^*} - \hbar\omega\right) \sum_{\lambda} |\boldsymbol{P}_i\cdot\hat{\boldsymbol{e}}^{(\lambda)}|^2. \tag{4.10}$$

The sum over the polarisation λ can be readily done using (3.7) to obtain

$$\sum_{\lambda=1,2} |\boldsymbol{P}_i\cdot\hat{\boldsymbol{e}}^{(\lambda)}|^2 = P_i^2 \sin^2\theta_k \tag{4.11}$$

where θ_k is the angle between $\boldsymbol{k}$ and $\boldsymbol{P}_i$. The subsequent steps are familiar, and they lead from (4.10) to the following expression:

$$\Gamma_{\mathrm{TO}}(E_i) = \frac{e^2\omega_{\mathrm{T}}}{4\hbar c^2 P_i} \int_{k-}^{k+} \mathrm{d}k \left[4\left(\frac{P_i^2}{m^*} - \hbar\omega_{\mathrm{T}}\right)\frac{1}{k} - \frac{\hbar^2 k}{m^*} - \frac{4m^*\omega_{\mathrm{T}}^2}{k^3}\right] \tag{4.12}$$

with

$$\hbar k_{\pm} = P_i \pm (P_i^2 - 2m^*\hbar\omega_{\mathrm{T}})^{1/2} \tag{4.13}$$

and we have set $\omega \simeq \omega_{\mathrm{T}}$ in the integrand of (4.10) (see the inset to figure 1). The integrations in (4.12) are elementary and it is convenient to express the result in terms of the dimensionless variable X

$$X = P_i^2/2m^*\hbar\omega_{\mathrm{T}} \equiv E_i/\hbar\omega_{\mathrm{T}}. \tag{4.14}$$

We find from (4.12)

$$\Gamma_{\mathrm{TO}}(X) = [2e^2\omega_{\mathrm{T}}^{3/2}/(2m^*\hbar)^{1/2}c^2]\{(2X^{1/2} - X^{-1/2})\sinh^{-1}[(X-1)^{1/2}] - (X-1)^{1/2}\}. \tag{4.15}$$

Equation (4.15) is the main result of this paper, giving the TO rate as a function of the initial electron energy. In its present form this result applies to any polar material for which the isotropic continuum model is appropriate as in III–V compounds.

It is instructive to compare the TO rate we have just derived with the other emission rates involving polar optical modes. We consider first the emission of LO modes via the Fröhlich interaction. The corresponding transition rate can be written in the following form

$$\Gamma_{\mathrm{LO}}(X) = \Gamma_0 \sinh^{-1}[(\xi X - 1)^{1/2}]/(\xi X)^{1/2} \tag{4.16}$$

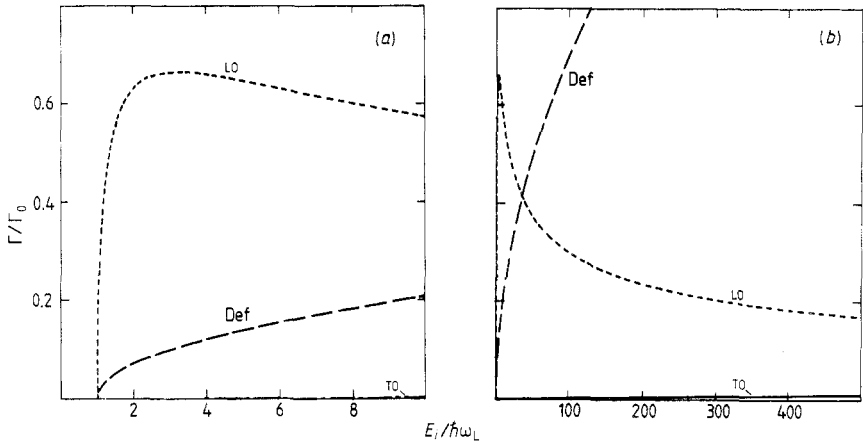

Figure 2. Transition rates Γ_{LO}, Γ_{TO} and Γ_{Def} against electron energy (in units of $\hbar\omega_L$) in GaAs: (*a*) in the low-energy region where Γ_{TO} is small; (*b*) in the higher-energy region. See the text for the assumed set of parameters.

where

$$\xi = (\varepsilon_0/\varepsilon_\infty)^{1/2} \tag{4.17}$$

$$\Gamma_0 = (e^2/\hbar\bar{\varepsilon})(2m^*\omega_L/\hbar)^{1/2} \tag{4.18}$$

with

$$\omega_L^2 = (\varepsilon_0/\varepsilon_\infty)\omega_T^2 \tag{4.19}$$

$$\bar{\varepsilon} = \varepsilon_0\varepsilon_\infty/(\varepsilon_0 - \varepsilon_\infty). \tag{4.20}$$

The result (4.16) is well known (see, e.g., Conwell 1967, Ridley 1988) and it has been derived recently using a fully quantised approach (Chamberlain 1987). The quantity Γ_0 in (4.18) characterises the LO rate since the function in (4.16) cannot exceed unity. It is therefore useful to write (4.15) in terms of Γ_0. We find

$$\Gamma_{TO}(X) = \Gamma_0(\varepsilon_0/\varepsilon_\infty)^{1/2}(\bar{\varepsilon}\hbar\omega_T/m^*c^2) \times \{(2X^{1/2} - X^{-1/2})\sinh^{-1}[(X-1)^{1/2}] - (X-1)^{1/2}\}. \tag{4.21}$$

Finally we consider energy relaxation via the optical deformation potential. This effect involves both LO and TO modes (see, e.g., Ridley 1988). The transition rate for emission by electrons can be written in the form

$$\Gamma_{Def} = (D_0^2/4\pi\rho\hbar^2)(2m_l^* m_t^{*2}/\hbar\omega_T)^{1/2}(X-1)^{1/2}. \tag{4.22}$$

Here D_0 is the optical deformation constant and ρ is the mass density. m_l^* and m_t^* are the longitudinal and transverse effective masses, respectively.

We compare the three emission rates Γ_{TO}, Γ_{LO} and Γ_{Def} for the case of GaAs. The parameters are $\varepsilon_0 \simeq 12.93$, $\varepsilon_\infty \simeq 10.9$, $\hbar\omega_L \simeq 0.036$ eV, $m^* \simeq 0.06\,m_e$ which yield for Γ_0

$$\Gamma_0 \simeq 7 \times 10^{12}\ \text{s}^{-1}. \tag{4.23}$$

Furthermore we have $m_l^* \simeq 1.9\,m_e$, $m_t^* \simeq 0.075\,m_e$, $D_0 \simeq 10^{11}$ eV m^{-1} and $\rho \simeq 5.36 \times 10^3$ kg m^{-3}. The results are shown in figures 2(*a*) and (*b*), which compare the variations of the three rates with varying electron energy $E_i/\hbar\omega_L$. It is clear that the emission rate arising from the electromagnetic coupling to the TO modes is practically

dominated by those due to the polar LO and the deformation potential mechanism in this material.

5. Comments and conclusions

As far as we know this paper represents the first attempt to quantify energy relaxation arising from the electromagnetic coupling of the TO modes to charge carriers in polar semiconductors. The appropriate theory has been developed here leading to the explicit evaluation of the corresponding emission rate Γ_{TO} as in equation (4.15). The result was compared in particular with those arising from other coupling mechanisms involving optical modes, namely, the Fröhlich interaction from the LO modes and the deformation potential involving both the LO and the TO modes. The results of the paper substantiate the suggestion which was always accepted on intuitive grounds that LO emission dominates energy relaxation via the Fröhlich mechanism except at high energies where the electromagnetic influence of the TO modes competes with that of the LO modes. On the other hand, in the high-energy region the contributions from the deformation potential dominate those due to the electromagnetic mechanisms. The high- and low-energy regions for GaAs can be inferred from figures 2(*a*) and (*b*).

The work presented in this paper provides the basis for the application of the theory to the case of low-dimensional structures where calculations of the various dynamical effects demand a re-examination of the factors contributing to the total relaxation rate. So far, attention seems to have been focused on how the LO rate is modified by the layering via modifications involving electronic band structure and LO waveforms. Similar considerations are clearly needed for the TO modes. This problem is now being investigated and the results will be given in a future report.

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